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Polynomial Form of the Stueckelberg Model ^{*}

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Abstract. - The Stueckelberg model for massive vector fields is cast into a BRS invariant, polynomial form. Its symmetry algebra simplifies to an abelian gauge symmetry which is sufficient to decouple the negative norm states. The propagators fall off like $1/k^2$ and the Lagrangean is polynomial but it is not powercounting renormalizable due to derivative couplings.

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The Stueckelberg model describes massive vector particles with gauge interactions. It has repeatedly attracted attention [1] as an alternative to the Higgs model and is particularly interesting as long as the Higgs particle is not confirmed experimentally. The model contains a set A_μ^a, φ^a of real vector and scalar fields ($a = 1, \dots, \dim(G)$) with a gauge invariant kinetic energy and with the interactions of a nonabelian gauge group G , which we take to be simple. The coupling constant g appears as a normalization $\frac{1}{g^2}$ in front of the action.

$$\mathcal{L}_{kin} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} \quad (1)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - f_{bc}^a A_\mu^b A_\nu^c \quad (2)$$

The mass term for the vector fields is introduced in a gauge invariant way.

$$\mathcal{L}_{mass} = -\frac{m^2}{g^2} \text{tr} (A_\mu - U^{-1} \partial_\mu U)^2 \quad (3)$$

where $U = U(\varphi)$ is an abbreviation for the series

$$U = e^{\frac{1}{m} \varphi^a T_a} \quad (4)$$

and A_μ is the matrix

$$A_\mu = A_\mu^a T_a. \quad (5)$$

The matrices T_a are a normalized basis of a matrix representation of the Lie algebra of G .

$$[T_a, T_b] = f_{ab}^c T_c \quad \text{tr} T_a T_b = -\frac{1}{2} \delta_{ab} \quad (6)$$

One easily confirms the invariance of \mathcal{L}_{kin} and of \mathcal{L}_{mass} under the gauge transformation generated by

$$V = e^{\lambda^a(x) T_a}. \quad (7)$$

$$U \rightarrow U' = U \cdot V \quad (8)$$

$$A_\mu \rightarrow A'_\mu = V^{-1} \cdot (A_\mu + \partial_\mu) \cdot V$$

In particular one can choose the gauge

$$V = U^{-1}. \quad (9)$$

Thereby one gets rid of the scalar fields φ^a and fixes the gauge symmetry and one may well wonder whether φ^a and the gauge symmetry with arbitrary functions λ^a are trivial. In the gauge (9) the Lagrangean for A'_μ

$$\mathcal{L}_{inv} = \mathcal{L}_{kin} + \mathcal{L}_{mass} \quad (10)$$

is the Lagrangean $\mathcal{L}_{YM, mass}$ of massive Yang Mills fields with a propagator

$$\langle TA'_\mu(x)A'_\nu(0) \rangle = -ig^2 \int \frac{d^4k}{(2\pi)^4} e^{ikx} \frac{\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}}{k^2 - m^2 + i\epsilon} \quad (11)$$

which does not fall off like $1/k^2$.

A different choice of the gauge, however, leads to well behaved propagators where $\langle AA \rangle$ and $\langle \varphi\varphi \rangle$ fall off like $1/k^2$. The additional propagating degrees of freedom in A and φ , which yield these improved propagators, have to be compensated by anticommuting Fadeev-Popov ghosts b^a and c^a . They decouple from the three physical spin-1 modes in the vector field because \mathcal{L}_{inv} and the gauge fixing and ghost Lagrangean are invariant under BRS transformations s [2]. Explicitly s is given by

$$\begin{aligned} s A_\mu^a &= \partial_\mu c^a - c^b A_\mu^c f_{bc}^a \\ s \left(e^{\frac{1}{m}\varphi^a T_a} \right) &= \left(e^{\frac{1}{m}\varphi^a T_a} \right) c^b T_b \\ s c^a &= -c^b c^c f_{bc}^a \\ s b^a &= B^a \\ s B^a &= 0. \end{aligned} \quad (12)$$

B^a is an auxiliary field introduced to make the BRS transformation s offshell nilpotent. The transformation of φ which follows from (12) is nonpolynomial

$$s \varphi^a = m c^a + \frac{1}{2} c^c \varphi^b f_{bc}^a + \dots \quad (13)$$

and also the Lagrangean is an infinite series because of terms such as

$$\mathcal{L}_{inv} = \dots - m^2 \text{tr} U^{-1} \partial_\mu U U^{-1} \partial^\mu U.$$

The transformation of the fields and the Lagrangean can be simplified considerably. It is well known [3] that if one is given a set of fields which transform under a compact group G one can redefine the fields¹ such that they consist of multiplets of tensor fields transforming linearly under a subgroup H and Goldstone fields which transform like the coset G/H under group multiplication. In the case at hand the scalar fields transform as Goldstone fields of G , i.e. $H = \mathbb{I}$, and the vector fields A_μ , which transform under the adjoint transformation of G , can be

¹ The perturbatively evaluated S -matrix is unchanged by this field redefinition, global transformation properties like existence of fixed points in the space of fields are normally not preserved.

replaced by G -invariant vector fields \hat{A}_μ .

$$\begin{aligned}\hat{A}_\mu &= U \cdot (A_\mu - U^{-1} \partial_\mu U) \cdot U^{-1} \\ &= A_\mu - \frac{1}{m} \partial_\mu \varphi^a T_a + O(A_\mu \varphi, \partial_\mu \varphi \varphi)\end{aligned}\quad (14)$$

The Lagrangean $\mathcal{L}_{inv}(A, \partial A, \varphi, \partial \varphi)$ actually depends only on \hat{A} and $\partial \hat{A}$ and only in a polynomial form

$$\begin{aligned}\mathcal{L}_{inv}(A, \partial A, \varphi, \partial \varphi) &= \mathcal{L}_{inv}(\hat{A}, \partial \hat{A}, 0, 0) \\ &= \mathcal{L}_{YM, mass}(\hat{A}, \partial \hat{A}).\end{aligned}\quad (15)$$

The ghost system is drastically simplified by a redefinition of the ghost fields c^a à la Brandt [4].

$$\hat{c}^a = \frac{1}{m} s \varphi^a \quad (16)$$

This is also an invertible field transformation which does not change the perturbatively evaluated S -matrix. It casts the transformation of the scalar fields into an abelian form with no remnants of the structure constants f_{ab}^c of the nonabelian group G . In terms of the fields \hat{A}_μ and \hat{c}^a the BRS transformations are linear and read

$$\begin{aligned}s \hat{A}_\mu &= 0 \\ s \varphi^a &= m \hat{c}^a \quad s \hat{c}^a = 0 \\ s b^a &= B^a \quad s B^a = 0.\end{aligned}\quad (17)$$

The BRS multiplets consist of pairs (φ^a, \hat{c}^a) , (b^a, B^a) and singlets \hat{A}_μ^a . Therefore all BRS invariant local actions, i.e. all Lagrangeans which are invariant up to derivatives, follow from the basic lemma (see e.g. [5]).

$$s\mathcal{L} = 0 \Leftrightarrow \mathcal{L} = \mathcal{L}_{phys}(\hat{A}, \partial \hat{A}) + s(b^a X_a), \quad (18)$$

where X_a are real bosonic functions of all the fields and their derivatives. They have to have ghost number 0 and can otherwise be conveniently chosen because the piece $s(b^a X_a)$ has no measurable effect on scattering amplitudes of physical states, as long as a change of parameters in X_a perturb these scattering amplitudes smoothly.

If, for example, we choose $X_a = X_a(\varphi, B) = \frac{1}{2} B^a - (\square + m^2) \frac{\varphi^a}{m}$ then φ^a are dipole fields [6]. They contain two species of creation operators for spin-0 particles. One of the creation operators, a^\dagger , appears with the usual plane wave e^{ikx} , it is not invariant under BRS transformation but transformed into the creation operator c^\dagger contained in \hat{c}^a , because $s\varphi = m \hat{c}$. Hence $sa^\dagger = m c^\dagger$. The other creation operator in φ^a , d^\dagger , appears with $x^0 e^{ikx}$. Since there is no $x^0 e^{ikx}$ term in \hat{c} , $s\varphi = m \hat{c}$ implies that $sd^\dagger = 0$. This agrees with $sb = B = (\square + m^2) \frac{\varphi}{m}$ because in $(\square + m^2)\varphi$ only

a term $2ik^0 d^\dagger e^{ikx}$ is left, while b contains a term $b^\dagger e^{ikx}$ so that $sb^\dagger \propto d^\dagger$, and thus indeed $sd^\dagger = 0$. The doublets (b^\dagger, d^\dagger) and (a^\dagger, c^\dagger) have opposite ghost numbers, and form a Kugo-Ojima quartet [7].

These quartets of creation operators in the fields φ, \hat{c}, b decouple from the space of physical states which is generated by the vector field \hat{A}_μ . \hat{A}_μ has the undesirable propagator (11) if $\mathcal{L}_{phys} = \mathcal{L}_{YM, mass}$ and if the functions X_a are chosen to be independent of \hat{A}_μ .

One is tempted to add to $\mathcal{L}_{YM, mass}$ a piece $\frac{\lambda}{2g^2}(\partial_\mu A^\mu)^2$ to obtain a propagator $\langle \hat{A}\hat{A} \rangle$ which falls off like $\frac{1}{k^2}$ for $|k^2| \gg \frac{m^2}{\lambda}$. But this is achieved only at the expense of a coupled negative norm state at mass $\frac{m}{\sqrt{\lambda}}$ [8]. The invariance of the action under the BRS transformations (17) does not guarantee the decoupling of this negative norm state because the term $\frac{\lambda}{2g^2}(\partial_\mu A^\mu)^2$ is not introduced as part of the gauge fixing and ghost sector $s(b^a X_a)$ of the Lagrangean.

One can, however, obtain the desired propagator for the massive vector field if one chooses

$$X_a = \frac{1}{g^2} \left(\frac{1}{2} B^a - \partial \hat{A}^a - (\square + m^2) \frac{\varphi^a}{m} \right). \quad (19)$$

This yields the following gauge fixing and ghost part

$$\begin{aligned} s(b^a X_a) &= \frac{1}{2g^2} (B^a - \partial \hat{A}^a - (\square + m^2) \frac{\varphi^a}{m})^2 - \\ &\quad - \frac{1}{2g^2} (\partial \hat{A}^a + (\square + m^2) \frac{\varphi^a}{m})^2 + \\ &\quad + \frac{1}{g^2} b^a (\square + m^2) \hat{c}^a. \end{aligned} \quad (20)$$

The field B^a is auxiliary with the algebraic equation of motion

$$B^a = \partial \hat{A}^a + (\square + m^2) \frac{\varphi^a}{m}. \quad (21)$$

The ghosts b^a and \hat{c}^a are free! The $\hat{A} - \varphi$ sector does not contain higher derivatives or dipoles because

$$\begin{aligned} \partial \hat{A} + (\square + m^2) \frac{\varphi}{m} &= \partial \left(\hat{A} + \frac{1}{m} \partial \varphi \right) + m \varphi \\ &= \partial \bar{A} + m \varphi \end{aligned} \quad (22)$$

where

$$\bar{A} = \hat{A} + \frac{1}{m} \partial \varphi. \quad (23)$$

If one writes the Lagrangean in terms of \bar{A} and φ one obtains finally

$$\mathcal{L}_{YM, mass}(\hat{A}) + s(b^a X_a) = -\frac{1}{4g^2} F_{\mu\nu}^2 +$$

$$\begin{aligned}
& + \frac{m^2}{2g^2} (\bar{A} - \frac{1}{m} \partial\varphi)^2 - \frac{1}{2g^2} (\partial\bar{A} + m\varphi)^2 + \\
& + \mathcal{L}_{ghost} + \mathcal{L}_{auxiliary} = \mathcal{L}_{(2)} + \mathcal{L}_{int} .
\end{aligned} \tag{24}$$

In particular the free Lagrangean $\mathcal{L}_{(2)}$ does not mix the fields \bar{A} and φ . Dropping complete derivatives it takes the form

$$\begin{aligned}
g^2 \mathcal{L}_{(2)} = & \frac{1}{2} \bar{A}_\mu^a (\square + m^2) \bar{A}^{\mu a} - \frac{1}{2} \varphi^a (\square + m^2) \varphi^a + \\
& + b^a (\square + m^2) \hat{c}^a + \frac{1}{2} (B^a - \partial\bar{A}^a - m\varphi^a)^2 .
\end{aligned} \tag{25}$$

All propagators fall off like $1/k^2$.

The states which are generated by $\partial\bar{A}$ decouple because they are not BRS invariant and the states generated by $\varphi + \frac{1}{m} \partial\bar{A} = s \frac{1}{m} b$ decouple because they are BRS trivial. Together with the states generated by b^a and \hat{c}^a they form BRS quartets [7] which decouple from the Hilbert space of physical states.

The interaction is contained in the interaction part of $\mathcal{L}_{YM,mass}$ where, however, the field \hat{A} is replaced by $\bar{A} - \frac{1}{m} \partial\varphi$.

$$\mathcal{L}_{int} = -\frac{1}{4g^2} (F_{\mu\nu}^a (\bar{A} - \frac{1}{m} \partial\varphi))^2|_{int} \tag{26}$$

It contains derivative couplings of φ up to dimension 8.

The nonrenormalizability from the infinite series $U(\varphi)$ has been tamed to a finite number of nonrenormalizable interactions and the Stueckelberg model has been simplified considerably.

In fact, it has been simplified so much that a sobering analysis of the physical amplitudes is possible. Physical states are generated by the spin-1 part of the field $\hat{A} = \bar{A} - \frac{1}{m} \partial\varphi$. This field has only interactions with itself, in fact it has just the Yang Mills interactions. The propagator of \hat{A} follows from the propagators of A and φ

$$\langle \hat{A} \hat{A} \rangle = \langle A A \rangle + \frac{1}{m^2} \langle \partial\varphi \partial\varphi \rangle \tag{27}$$

and turns out to be just the one in (11) we started from.

$$\langle T \hat{A}_\mu(x) \hat{A}_\nu(0) \rangle = -ig^2 \int \frac{d^4 k}{(2\pi)^4} e^{ikx} \frac{\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}}{k^2 - m^2 + i\epsilon} \tag{28}$$

In the polynomial formulation of the Stueckelberg model the propagator with bad high energy behaviour is exchanged for well behaved propagators and derivative couplings. While such a reformulation is mathematically completely equivalent it may nevertheless be more fruitful to think of the diagrams of massive Yang Mills

theory in terms of exchanged vectors and scalars. For example, when Veltman analyzed massive Yang Mills theories [9] he could show the absence of on-shell one loop divergencies in 1PI diagrams with more than 4 external spin-1 particles. He achieved his result by adding and mixing free scalars in a very similar manner to the approach followed here. However, he only claimed that the one-loop theory was power-counting renormalizable, not that it was multiplicatively renormalizable [10]. At the two-loop level, he showed that replacing k^2 by $k^2 + m^2$ in YM theory, keeping the rest of the Feynman rules as in the massless case, leads to non-unitarity. He also studied which contact terms are needed for unitarity (noting that $(\eta_{\mu\nu} - \partial_\mu \partial_\nu / m^2)(\Theta \Delta^{+/-})$ differs from the required $\Theta(\eta_{\mu\nu} - \partial_\mu \partial_\nu / m^2)\Delta^{+/-}$ by contact terms $\delta_{\mu 0} \delta_{\nu 0} \delta^3(x)$). However, he reached no definite conclusions at the two-loop level [11]. We stress that renormalization of off-shell Greens functions can be weakened to renormalization of S-matrix elements. It would be a formidable but interesting problem to analyze whether massive YM theory has a two-loop renormalizable S-matrix.

It is interesting to know whether by field redefinitions one can cast a model, like the polynomial Stueckelberg model, into a power counting renormalizable form. In the massive, abelian case this is possible, because the field strengths $F_{\mu\nu}$ calculated from \bar{A} and from $\bar{A} - \frac{1}{m}\partial\varphi$ coincide and the model is actually free. If one introduces additional matter, like a gauge invariant Dirac fermion Ψ , and adds a gauge invariant interaction

$$\mathcal{L}_{matter} = \bar{\Psi}(i\gamma\partial - M)\Psi + \bar{\Psi}\gamma(\bar{A} - \frac{1}{m}\partial\varphi)\Psi \quad (29)$$

then the apparent nonrenormalizable derivative coupling $\bar{\Psi}\gamma\partial\varphi\Psi$ can be absorbed by a field redefinition

$$\psi = e^{\frac{i}{m}\varphi}\Psi, \quad (30)$$

$$\mathcal{L}_{matter} = \bar{\psi}(\gamma(i\partial + \bar{A}) - M)\psi. \quad (31)$$

In the massive, nonabelian case this question was first analyzed in [12]. It was shown that the criterion of tree-unitarity uniquely leads to the Higgs model. (See also [13] where it was shown using asymptotic BRS-invariance that the nonabelian Stueckelberg model fails to be power-counting renormalizable and that no field redefinitions exist to establish this property.) However, it would be nice to determine by algebraic methods only whether for the self interacting nonabelian theory described by the Stueckelberg model a renormalizable formulation is possible only if one introduces additional scalar fields and additional couplings of the scalar fields such that the Stueckelberg model becomes a subsector of the nonabelian Higgs model.

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